Efficient outdoor sound propagation modelling in time-domain

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CEAS, X-Noise : <u>Atm. and ground effects on aircraft noise</u> – Sevilla, 2013

Why time-domain models?

- Sound propagation is essentially a time-domain process
- Response over broad frequency range possible with a single run
- Including of non-linear effects
- Modelling realistic sources (moving, transient)
- Fluid-flow acoustics coupling can be treated more easily

Outline

- **Finite-difference time-domain (FDTD) method**
  - Solving LEE
  - Numerical discretisation strongly influences modelling efficiency
    - **computational cost**
    - **numerical accuracy**
    - **numerical stability**
  - In absence/presence of flow
  - Finite absorbers
- **Long distance sound propagation**
  - moving frame approach
  - hybrid modelling

Sound propagation equations

- **Linear continuous sound propagation equation in still air**
  - **Navier-Stokes equations reduced to**
    - Momentum equation (*velocity equation*)
    - Continuity equation (*pressure equation*) + (linear) pressure-density relation
  - **Assumptions**
    - Linearization in acoustical quantities
    - Non-moving propagation medium
    - No thermal, viscous effects and molecular relaxation
    - No gravity

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho_0}\nabla p = 0$$

$$\frac{\partial p}{\partial t} + c^2 \rho_0 \nabla \cdot \mathbf{v} = 0$$

$$\frac{\partial^2 p}{\partial t^2} - c^2 \nabla^2 p = 0$$

$$p_{idx,jdy,kdz}^{ldt} = p_{i,j,k}^{l}$$

Lowest possible spatial stencil

- Two options for central differences
  - **Collocated-in-place (CIP)**
  - **Staggered-in-place (SIP)**

$$p_{i+1,j,k} = p_{i,j,k} + \frac{dx}{1!}\frac{\partial p}{\partial x}\bigg|_{i,j,k} + \frac{dx^2}{2!}\frac{\partial^2 p}{\partial x^2}\bigg|_{i,j,k} + \frac{dx^3}{3!}\frac{\partial^3 p}{\partial x^3}\bigg|_{i,j,k} + \dots$$

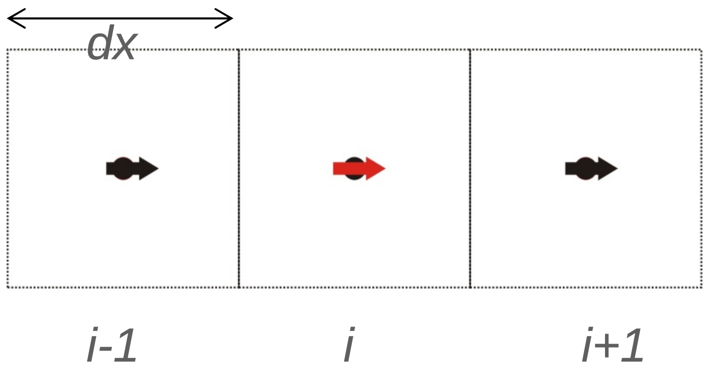
$$p_{i-1,j,k} = p_{i,j,k} - \frac{dx}{1!}\frac{\partial p}{\partial x}\bigg|_{i,j,k} + \frac{dx^2}{2!}\frac{\partial^2 p}{\partial x^2}\bigg|_{i,j,k} - \frac{dx^3}{3!}\frac{\partial^3 p}{\partial x^3}\bigg|_{i,j,k} + \dots$$

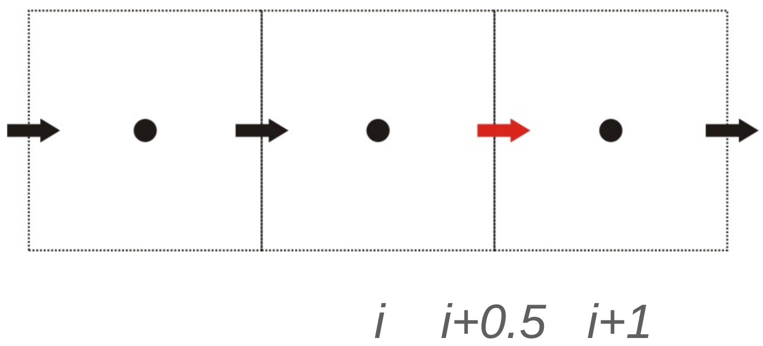
$$\frac{\partial p}{\partial x}\bigg|_{i,j,k} = \left(\frac{p_{i+1,j,k} - p_{i-1,j,k}}{2dx}\right) - \frac{dx^2}{3!}\frac{\partial^3 p}{\partial x^3}\bigg|_{i,j,k} + \dots$$

$$p_{i+1,j,k} = p_{i+0.5,j,k} + \frac{dx}{2.1!}\frac{\partial p}{\partial x}\bigg|_{i+0.5,j,k} + \frac{dx^2}{2^2.2!}\frac{\partial^2 p}{\partial x^2}\bigg|_{i+0.5,j,k} + \frac{dx^3}{2^3.3!}\frac{\partial^3 p}{\partial x^3}\bigg|_{i+0.5,j,k}$$

$$p_{i,j,k} = p_{i+0.5,j,k} - \frac{dx}{2.1!}\frac{\partial p}{\partial x}\bigg|_{i+0.5,j,k} + \frac{dx^2}{2^2.2!}\frac{\partial^2 p}{\partial x^2}\bigg|_{i+0.5,j,k} - \frac{dx^3}{2^3.3!}\frac{\partial^3 p}{\partial x^3}\bigg|_{i+0.5,j,k}$$

$$\frac{\partial p}{\partial x}\bigg|_{i+0.5,j,k} = \left(\frac{p_{i+1,j,k} - p_{i,j,k}}{dx}\right) - \frac{dx^2}{4.3!}\frac{\partial^3 p}{\partial x^3}\bigg|_{i+0.5,j,k} + \dots$$





FDTD spatial aspects

Extended spatial stencil

- Take more neighbouring cells to better approach the gradient
  - **E.g. Involve 6 neighbouring cells (7-point stencil, CIP, central differences)**

$$\left.\frac{\partial p}{\partial x}\right|_{x_0} = \frac{p(x_0+3dx)-9p(x_0+2dx)+45p(x_0+dx)-45p(x_0-dx)+9p(x_0-2dx)-p(x_0-3dx)}{60dx}+O(dx^6)$$

a_i	Taylor	DRP
$a_{-3}=-a_3$	-0.0167	-0.0208
$a_{-2}=-a_2$	0.1500	0.1667
$a_{-1}=-a_1$	-0.7500	-0.7709
a_0	0	0

  - **Dispersion-relation preserving (DRP) schemes**
    - Numerically optimize values of a_i to further decrease phase error

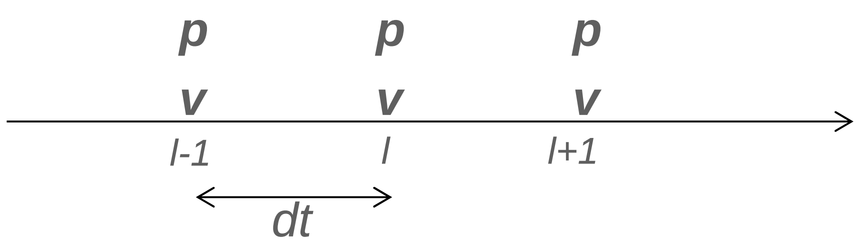
$$\left.\frac{\partial p}{\partial x}\right|_{x_0} \cong \frac{1}{dx}\sum_{i=-n}^{n} a_i p(x_0+idx)$$

- Drawbacks
  - **Point source representation difficult**
  - **Complicated boundary treatment**
  - **Reduced time steps for numerical stability**

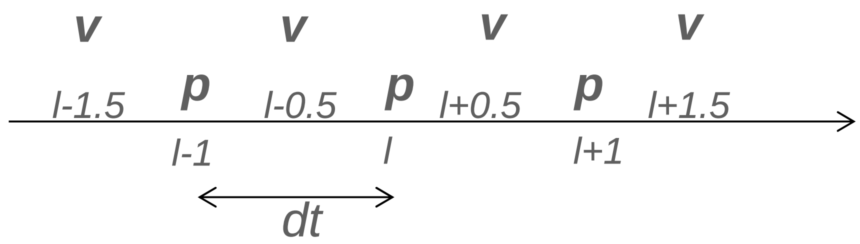
FDTD temporal aspects

■ Lowest-order schemes

- Two options for explicit schemes
  - **Collocated-in-time (CIT)**
  - **Staggered-in-time (SIT)**



$$p^{l} = p^{l-2} - 2dtc^{2}\rho_{0}\nabla\cdot\mathbf{v}^{l-1}$$



$$p^{l} = p^{l-1} - dtc^{2}\rho_{0}\nabla\cdot\mathbf{v}^{\text{l-0.5}}$$

- SIT is advantageous
  - **Higher numerical accuracy with lowest order central difference scheme (see spatial discretisation)**
  - **Halves memory use compared to CIT (in-place computation possible)**
  - **Doubles time step compared to CIT (stability)**

FDTD temporal aspects

- **High-order schemes**
  - By Taylor expansion
    - **In general : improves accuracy**
    - **Strongly increases memory cost**
  - Advanced schemes
    - **Runge-Kutta**
    - **Crank-Nickolson**
      - Implicit scheme
      - Stability guaranteed

Numerical stability

- Time-delay system
  - **Update equations can be written as a discrete time-delay system (z-transform of SIT scheme)**

$$\begin{cases} MP^{l} = M_{-1}P^{l-1} + \sum_{i} R_i V_i^{l-0.5} \\ N_i V_i^{l-0.5} = N_{i,-1.5} V_i^{l-1.5} + S_i P^{l-1} \end{cases}$$

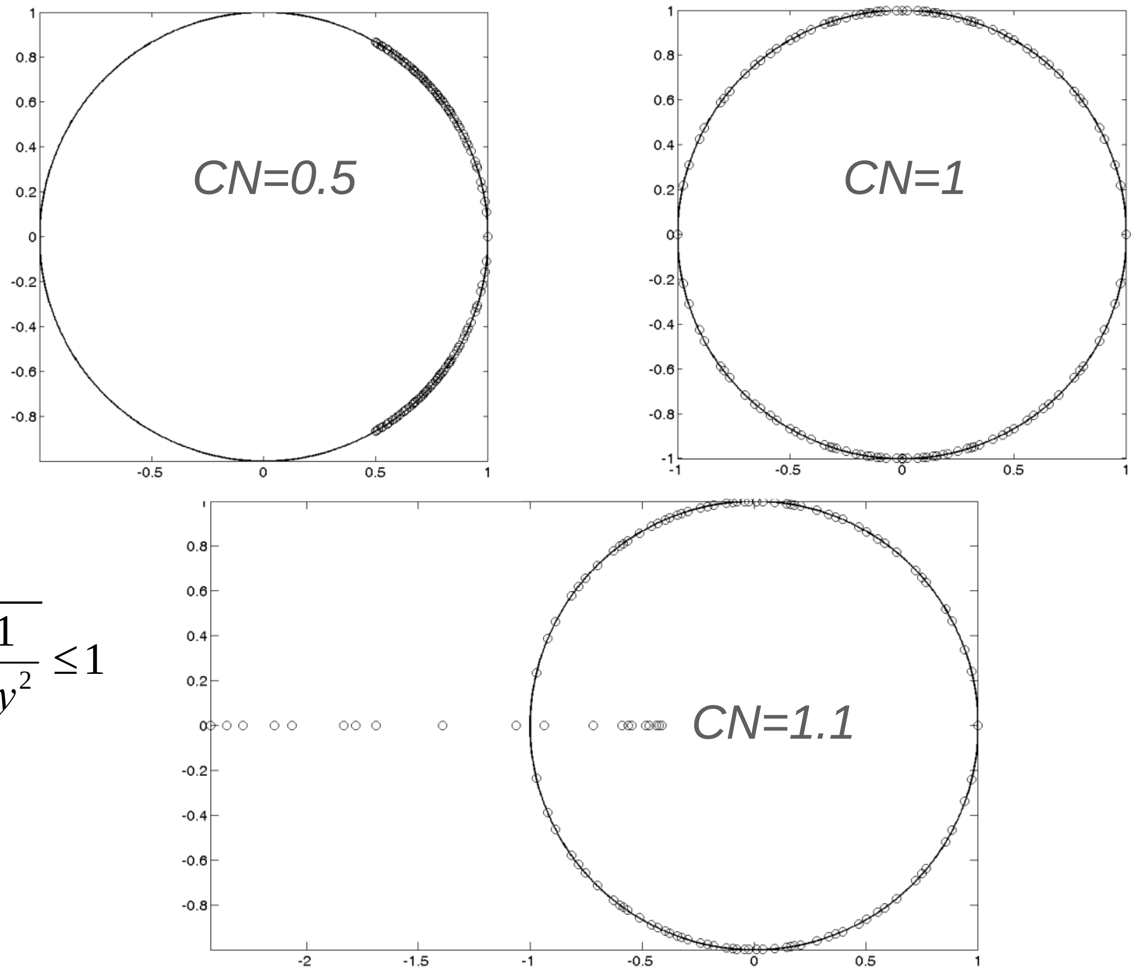
$$X^{m+1} = AX^{m}$$

$$X^{l} = \begin{bmatrix} P^{l} \\ V_i^{l-0.5} \end{bmatrix} \qquad A = \begin{bmatrix} M^{-1}M_{-1} + M^{-1}R_i N_i^{-1} S_i & M^{-1}R_i N_i^{-1} N_{i,-1.5} \\ N_i^{-1} S_i & N_i^{-1} N_{i,-1.5} \end{bmatrix}$$

  - **Poles of system should have a modulus smaller than or equal to one (or abs of eigenvalues of A should be smaller than 1)**

Numerical stability

- Time-delay system : pole plots (SIT,SIP)



$$CN = cdt\sqrt{\frac{1}{dx^2}+\frac{1}{dy^2}} \leq 1$$

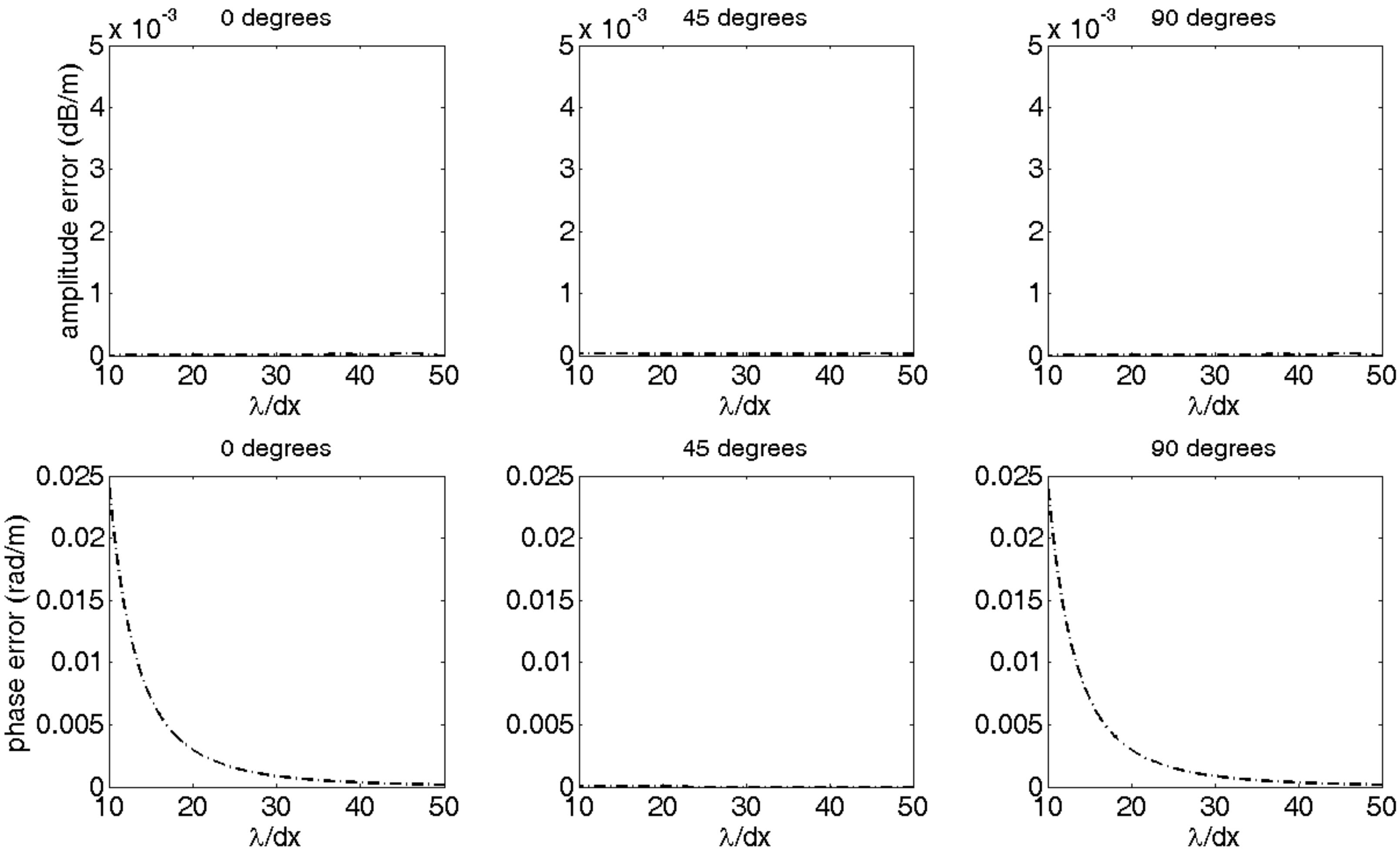
Numerical accuracy

- Two aspects
  - **Phase error**
  - **Amplitude error**
- SIP/SIT p-v FDTD is amplitude-error free
  - **At all Courant Numbers**
- FDTD results in phase errors
  - **Phase error decreases with finer spatial discretisation**
  - **Phase error vanishes when**
    - – CN=1
    - – Propagation along the diagonal of square cells

$$\Delta\varphi = 2\arcsin\left(c\,dt\sqrt{\frac{\sin^2(k_x dx/2)}{dx^2} + \frac{\sin^2(k_y dy/2)}{dy^2} + \frac{\sin^2(k_z dz/2)}{dz^2}} \right)$$

FDTD accuracy

Numerical accuracy



time-domain in OSP

Including meteorological effects

- Concept of “background flow”
  - **Most relevant interactions between wind and acoustics in outdoor applications near the ground included**
    - – Convection in uniform flows
    - – Refraction in non-uniform flows
    - – Scattering of sound
  - **No generation of sound**
  - **The acoustics will not influence the macro-fluid flow**

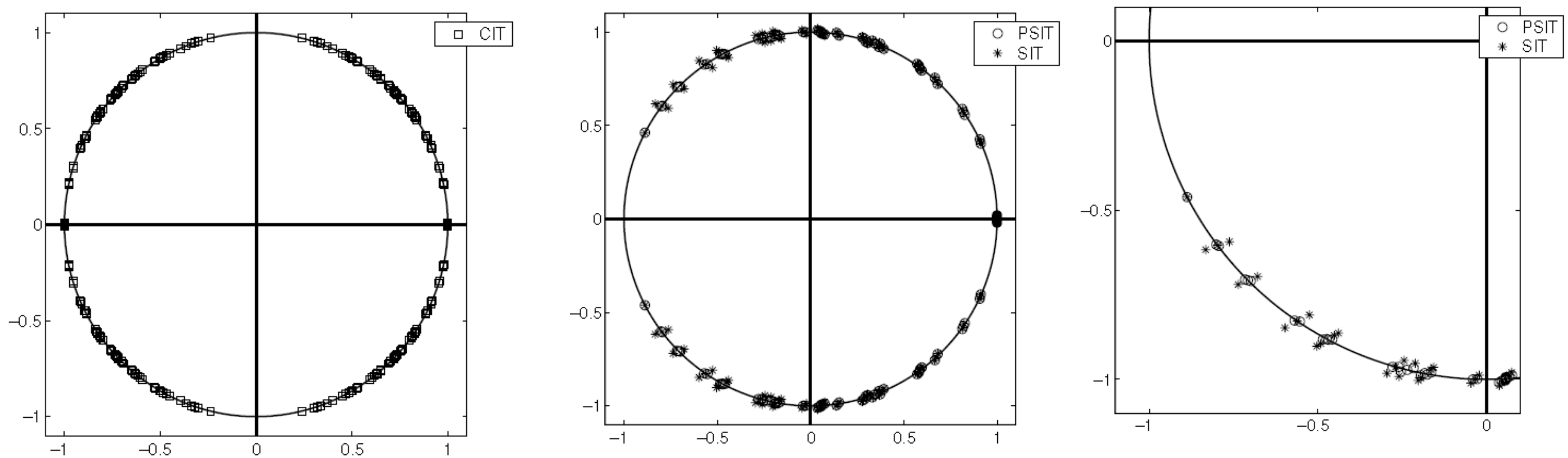
$$\frac{\partial p}{\partial t} + c^2 \rho_0 \nabla \cdot \mathbf{v} + \boldsymbol{v}_0 \cdot \nabla p = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\boldsymbol{v}_0 \cdot \nabla\right)\boldsymbol{v} + \left(\boldsymbol{v} \cdot \nabla\right)\boldsymbol{v}_0 + \frac{1}{\rho_0}\nabla p = 0$$

- Inhomogeneous atmosphere
  - **No additional cost**
  - **Highest sound speed determines stability criterion**

Numerical discretisation

- Spatial discretisation : SIP still interesting
- Temporal discretisation needs care
  - **SIT is moderately unstable**
  - **CIT is fully stable but computationally costly**
  - **PSIT weakly unstable**



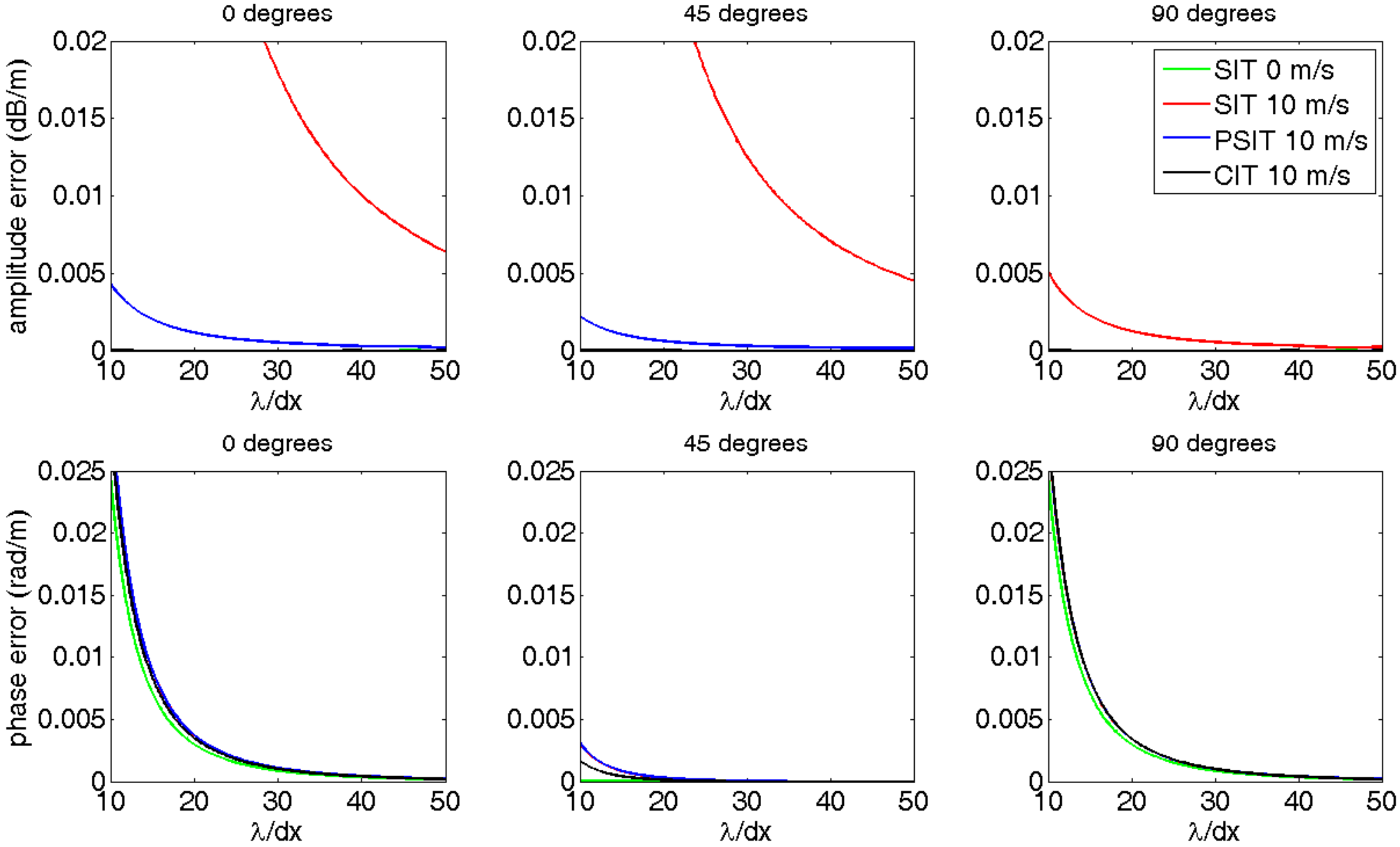
Numerical discretisation

- **PSIT scheme**
  - Second order terms in the flow speed are neglected during discretisation as wind speeds are typically low
  - Explicit, efficient scheme still possible
  - Numerical error
    - » Small amplitude-errors appear
    - » Phase error is not affected
  - see Van Renterghem et al.(Appl. Acoust., 2007)
  - Can be efficiently implemented

$$p^{l} = p^{l-1} - dtc^{2}\rho_{0}\nabla\cdot\boldsymbol{v}^{l\text{-}0.5} - dt\boldsymbol{v}_{\mathbf{0}}\cdot\nabla p_{noflow}^{l-0.5}$$

$$\boldsymbol{v}^{l+0.5} = \boldsymbol{v}^{l\text{-}0.5} - dt\frac{1}{\rho_{0}}\nabla p^{l} - dt\left(\boldsymbol{v}_{\mathbf{0}}\cdot\nabla\right)\boldsymbol{v}_{noflow}^{l} - dt\left(\boldsymbol{v}_{noflow}^{l}\cdot\nabla\right)\boldsymbol{v}_{\mathbf{0}}$$

Numerical discretisation



Finite absorbers

- **Impedance boundary condition**
  - Locally reacting surfaces
  - Models reflection at surfaces only
- **Including a second medium in the simulation domain**
  - Extended reaction (non-local reaction)
  - Models both reflection at surfaces, absorption inside, and transmission through materials
  - Spatially heterogeneous materials

Direct convolution

- Frequency domain impedance definition:

$$P(\omega) = Z(\omega) \cdot V(\omega)$$

- Each frequency domain signal or function has a time domain analogy $Z(t) = \Im^{-1}\{Z(\omega)\}$
- In time domain, we need a convolution which is a computationally costly operation

$$p(t) = Z(t) * v(t)$$

$$p(t) = \int_{-\infty}^{t} Z(t-\tau) v(\tau)\, d\tau = \int_{0}^{t} Z(t-\tau) v(\tau)\, d\tau$$

Impedance boundary cond.

- **Using exp. decaying time-domain functions**
  - Efficient direct convolution
  - Recursive approach

$$Z(\omega) = \frac{a}{1 + j\omega t_0} \qquad Z(t) = \frac{a}{t_0} e^{-\frac{t}{t_0}}$$

- **Series in jω**
  - Easy time-domain equivalent
  - Mass-spring-damper system

$$Z(\omega) = a_{-1}\frac{1}{j\omega} + a_0 + a_1 j\omega$$

$$p(t) = a_{-1}\int_{-\infty}^{t} v(t)\,dt + a_0 v(t) + a_1 \frac{dv(t)}{dt}$$

- **Pade approximants**
  - Examples of application
    - **Attenborough 4-parameter model**
    - **modified Zwikker and Kosten model**
- **Digital filters**
  - Efficient IIR filters
  - Highly flexibility to approach any w-Z curve

$$Z(z) = \frac{\sum_{i=0}^{n} a_i z^{-i}}{\sum_{i=0}^{m} b_i z^{-i}}$$

- **Poro-rigid frame model : Zwikker and Kosten**
  - Only the air in between the material matrix is allowed to vibrate
  - Reasonable when density of the frame and the stiffness is significantly larger than those of air
  - 3-parameter model
    - σ **(flow resistivity)**
    - φ **(porosity)**
    - k_s **(structure factor)**

$$\nabla \cdot p + \frac{\rho_0 k_s}{\varphi}\frac{\partial \mathbf{v}}{\partial t} + \sigma \mathbf{v} = \mathbf{0}$$

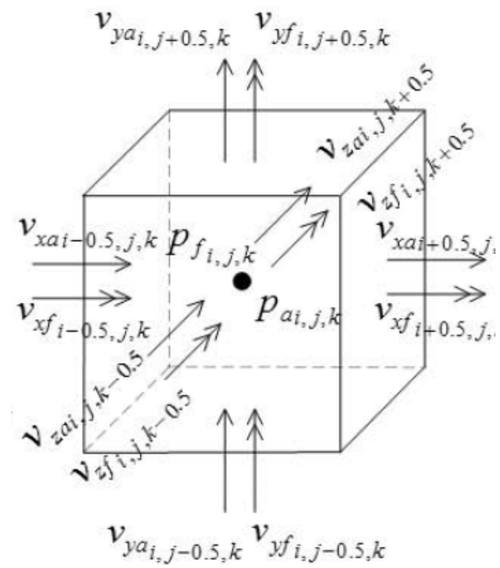
$$\frac{\partial p}{\partial t} + \frac{\rho_0 c_0^2}{\varphi}\nabla \cdot \mathbf{v} = 0$$

$$\frac{Z(\omega)}{Z_0} = \sqrt{\frac{\sigma}{\rho_0 \omega \varphi} j + \frac{k_s}{\varphi^2}}$$

Including porous medium

■ Poro-elastic models : M.A. Biot

- Coupled movement of frame and air inside the porous medium included
- slightly adapted version
- Parameters
  - **Tortuosity: m_t**
  - **Porosity: φ_a , $\varphi_f = 1-\varphi_a$**
  - **Flow resistivity: σ**
  - **Bulk modulus of frame: K_f**
  - **Frame density: ρ_f**
  - **Frame damping coefficient: R_f**



$$-\frac{\partial p_a}{\partial t} = K_a \varphi_a \nabla\cdot\mathbf{v}_a + \left(K_a - P_0\right)\varphi_f \nabla\cdot\mathbf{v}_f$$

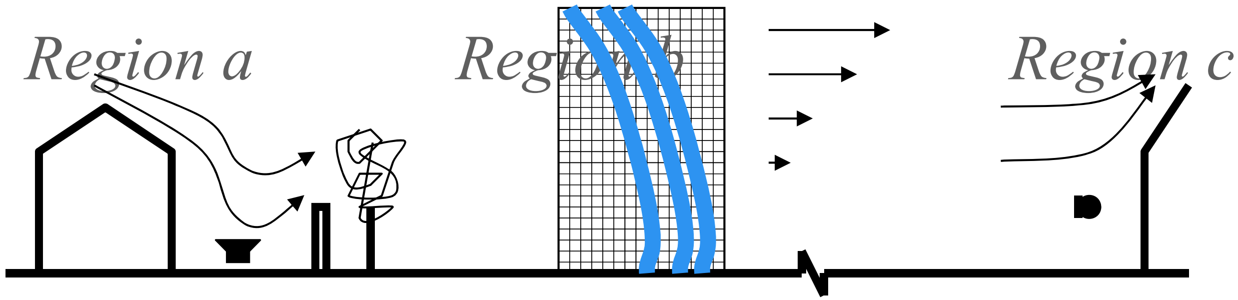
$$-\frac{\partial p_f}{\partial t} + \frac{\varphi_f}{\varphi_a}\frac{\partial p_a}{\partial t} = K_f \nabla\cdot\mathbf{v}_f$$

$$\rho_a \frac{\partial \mathbf{v}_a}{\partial t} = -\nabla p_a - \sigma\left(\mathbf{v}_a-\mathbf{v}_f\right) - \rho_a\left(\frac{m_t}{\varphi_a^{2}}-1\right)\frac{\partial}{\partial t}\left(\mathbf{v}_a-\mathbf{v}_f\right)$$

$$\rho_f \frac{\partial \mathbf{v}_f}{\partial t} + R_f\mathbf{v}_f = -\nabla p_f + \sigma\left(\mathbf{v}_a-\mathbf{v}_f\right) + \rho_a\left(\frac{m_t}{\varphi_a^{2}}-1\right)\frac{\partial}{\partial t}\left(\mathbf{v}_a-\mathbf{v}_f\right)$$

Long distance propagation

- Volume discretisation techniques not well suited for long-distance propagation
- Solutions
  - **Moving-frame FDTD**
    - – Use of short, broadband pulses
    - – Mainly software challenge
      - » Allocate and de-allocate memory in an efficient way
    - – Mainly efficient if propagation is essentially in one-direction

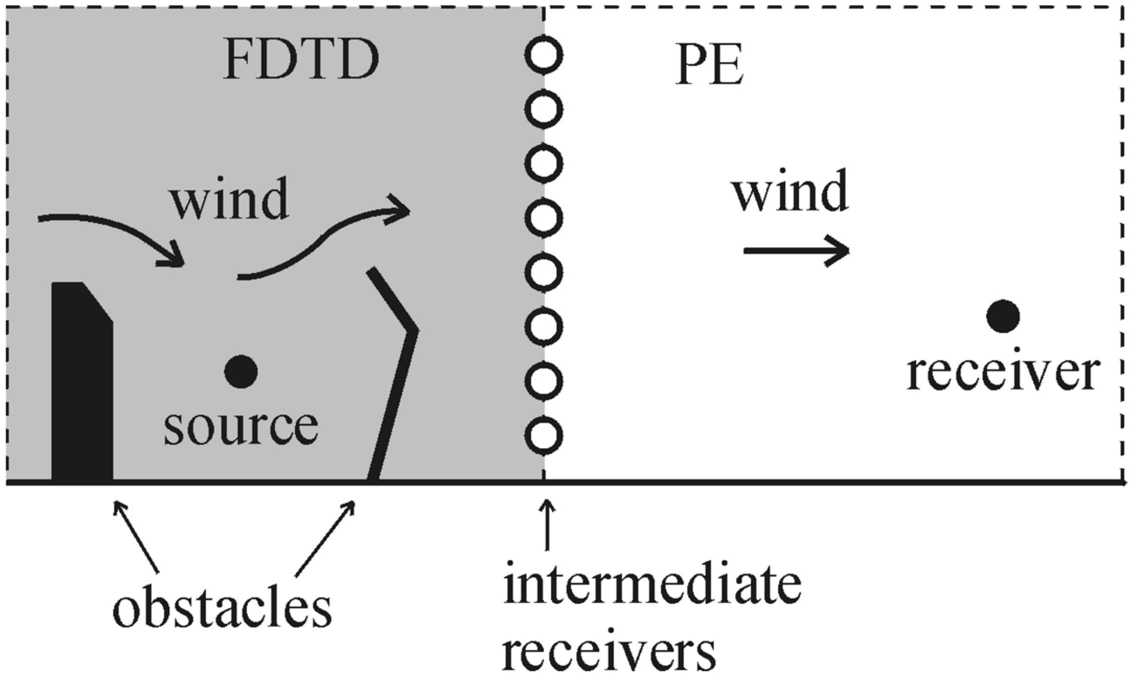


- **Long distance propagation**
  - Solutions
    - **Hybrid modelling**
      - Many techniques highly efficient in particular cases
      - Coupling in an attempt to combine “best of both worlds”
      - Coupling in same domain
        - BEM-PE
      - Cross-domain coupling
        - Raytracing-analytical formulae (e.g. diffraction)
        - BEM-raytracing
        - FDTD-PE

Long distance propagation

- Green’s Function Parabolic Equation method
  - **PE-type model : one-way sound propagation, effective sound speed approach, range-dependent impedance and profiles, inclusion of terrain possible (CMM,GTPE,rGFPE), diffraction over thin screen, etc.**
  - **Works with vertical array of acoustic pressures**
  - **Extrapollation towards next array based on Green’s function**
  - **Discretisation**
    - in vertical direction : strong discretisation
    - in forward direction : stepping at several wavelengths without loss in accuracy
  - **Efficiently uses FFT**

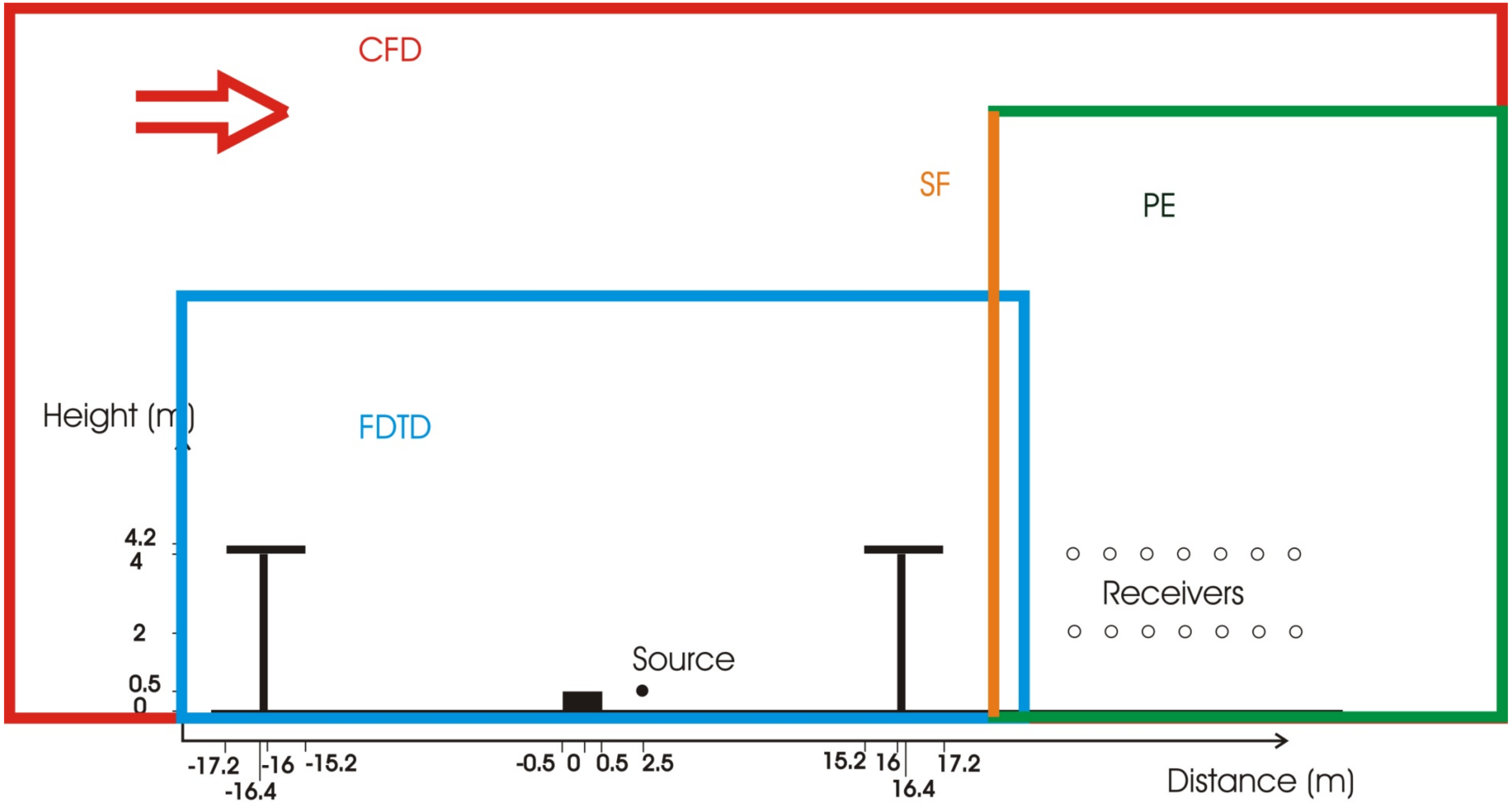
- **Long distance propagation**
  - FDTD-GFPE
    - **Typical road traffic noise application**
    - **Complex source region, less complex receiver zone**



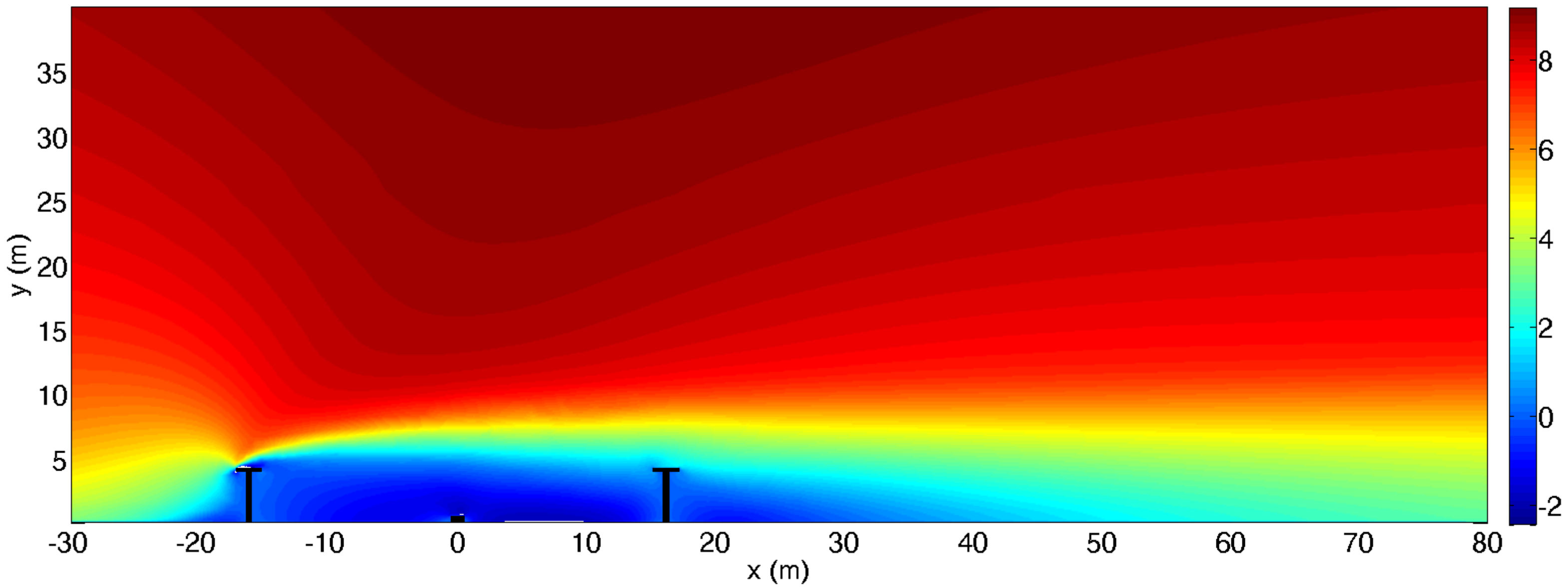
- **Long distance propagation**
  - FDTD-PE
    - **One-way coupling**
    - **Interface**
      - vertical array of receivers in FDTD
      - Time signal at each receiver recorded
      - Fourier transform gives starting function for PE for all frequencies of interest
      - Should be well chosen
    - **1FDTD, multiple PE calulations**

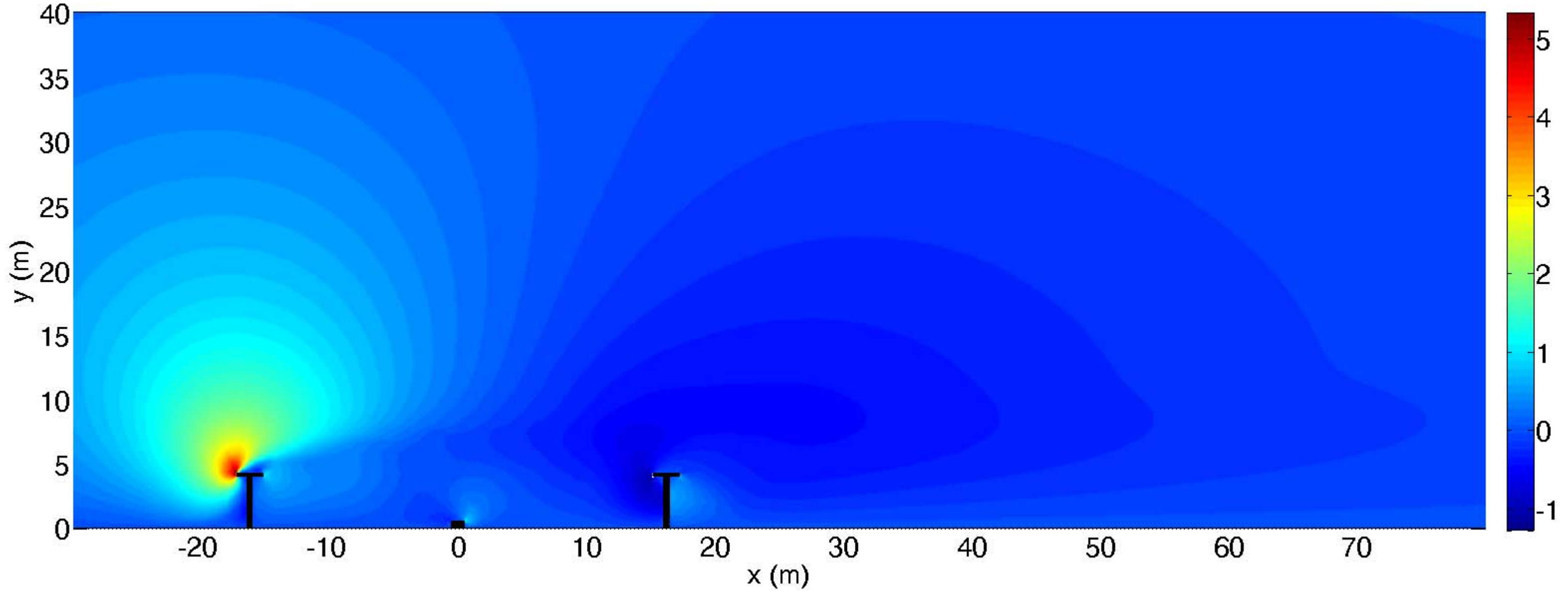
Long distance propagation

- FDTD-PE
  - **Example : evaluation of T-noise barriers in wind**



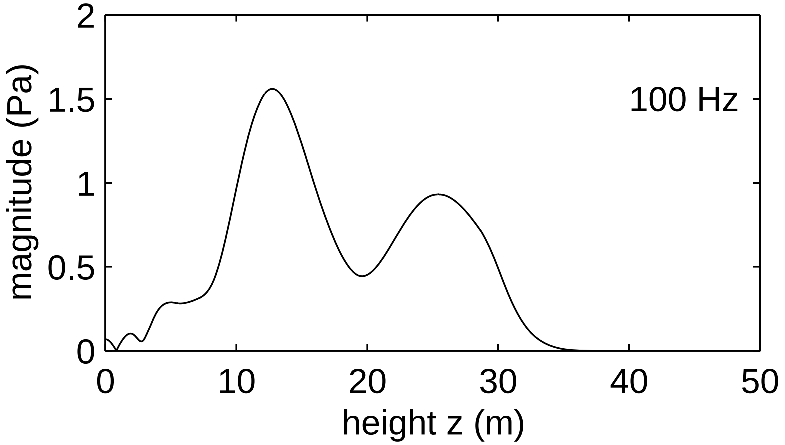
Hybrid modelling

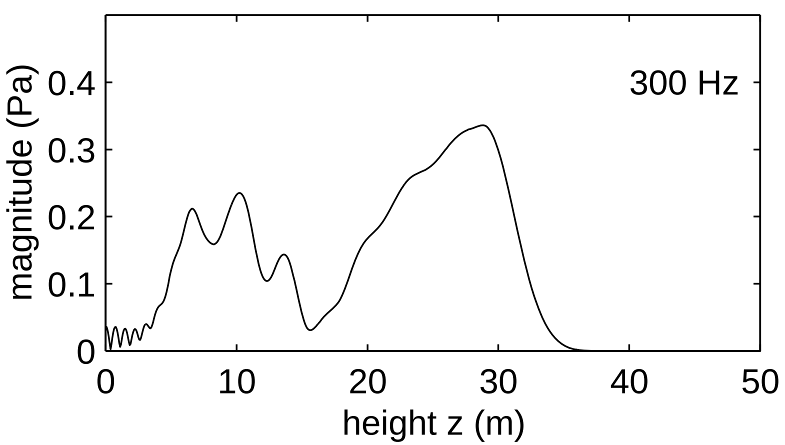


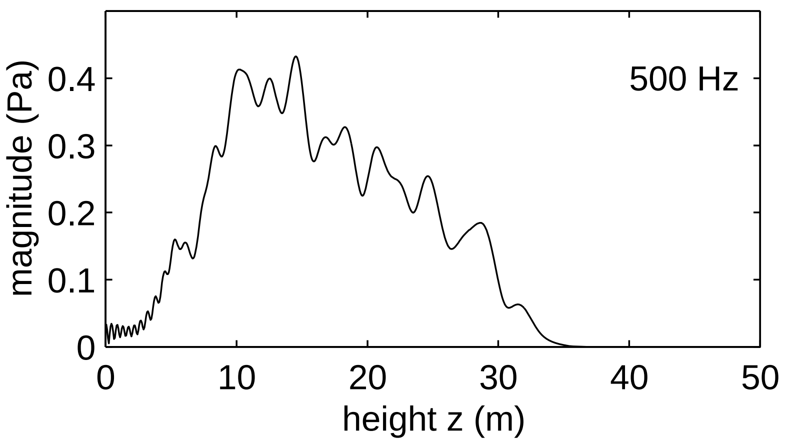


Long distance propagation

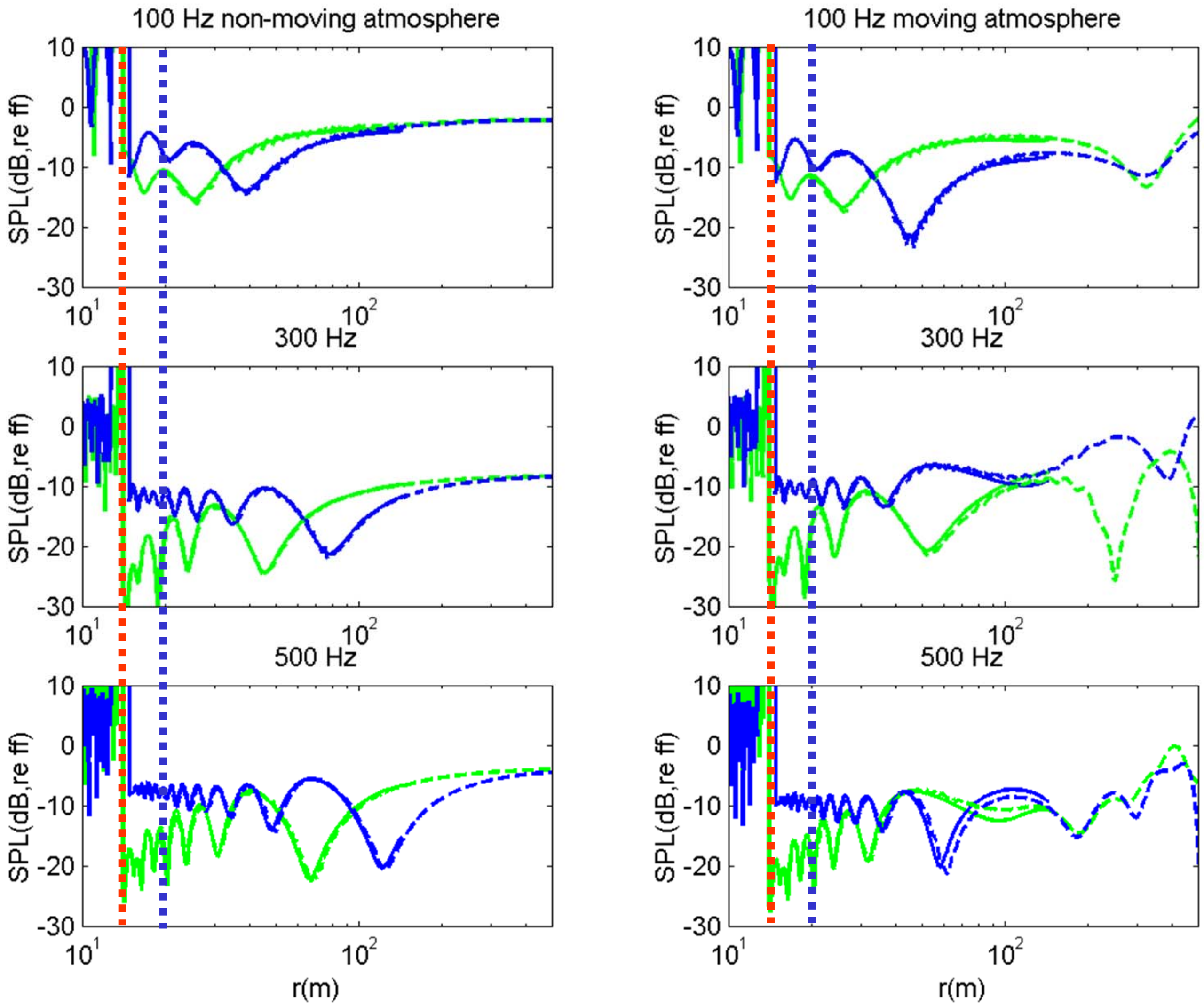
- FDTD-PE
  - **Starting fields**







Hybrid modelling



location downwind noise barrier

location starting field

FDTD 2 m

FDTD 4 m

FDTD+PE 2 m

FDTD+PE 4 m

Conclusions

- **Time-domain modelling in outdoor sound propagation has become mature**
- **Low-order schemes are well suited in outdoor sound propagation when carefully choosing the numerical discretisation scheme**
- **Hybrid modelling for long-distance sound propagation**
- **Current trends in time-domain modelling**
  - Parallellisation (by using GPU)
  - Pseudo-spectral time-domain technique (PSTD)